# THEORY AND MEASUREMENTS OF ANGLE-OF-ARRIVAL OF DIFFRACTION-LIMITED ELECTROMAGNETIC WAVE BEAMS IN THE TURBULENT ATMOSPHERE

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## ABSTRACT

Improvements in radar tracking performance based on the use of interferometric and distributed aperture radar systems, together with the need for command-guided weapons for cost reduction, have placed more stringent requirements on radar angular accuracy. The result of this upgraded accuracy requirement is that fluctuations in angle-of-arrival (AOA) due to small-scale atmospheric turbulence become significant in the error budgets of command-guided weapons systems. In this paper we present a theory of AOA variations based on the incorporation of diffraction-limited Gaussian beam propagation into a geometrical optics formulation of AOA. Measurements of AOA are compared to this theory with good agreement for this type of experiment.

## 1. BACKGROUND

The effect of atmospheric turbulence on radar angle errors is generally small, but in those cases where a radar is required to guide a missile without a seeker (command guide), a few microradians of error may be enough to cause the missile to miss its target. In this paper we compare two methods of calculating these AOA errors and show that the geometrical optics approach derived by Churnside and Lataitis [1] gives larger angular errors than those determined by experiment. This method is characterized by perfectly collimated beams and ideal plane waves. The second method involves the adaptation of the approach described in [1] to a physical optics mode in which the beams have a Gaussian profile. This method of calculation gives results close to experimental values. We determine the AOA using both approaches and compare the results obtained to an experiment conducted using an interferometric radar capable of measuring the small angles expected for this scenario [2].

## 2. THEORY

In deriving the AOA for the physical optics case, we proceed exactly as for the geometrical optics case using the procedure given in [1]. For one-way transmission, we have shown [3] that the variance of the AOA measured at the target is,

$$\sigma_t^2 = 2.92 \frac{C_n^2}{w^2(L)} \int_0^L [w(z_1^2)]^{5/3} dz_1 \tag{1}$$

where w(L) and $w(z_1)$ are the beamwidths measured at L and $z_1$, respectively. In the physical optics case, the beam has a Gaussian profile and its width varies as [4]:

$$w(z) = 2w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}, \tag{2}$$

where λ is wavelength and $w_0$ is the 1/e beamwidth at the transmitter. For a reflected beam, this approach is extended to 2-way transmission using the method described in [1]. The result for the AOA variance back at the radar receiver is: (3)

$$\sigma_r^2 = \frac{C_n^2}{w_r^2(0)} \begin{pmatrix} 2.92\left[ \int_0^L w_t^{5/3}(z_1) dz_1 + \int_0^L w_r^{5/3}(z_1) dz_1 \right] \\ + 2^{1/3} L \int_0^L \{ [w_t(z_1) + w_r(z_1)]^{2/3} - |w_t(z_1) - w_r(z_1)|^{2/3} \} dz_1 \end{pmatrix}$$

where $w_r(0)$ is ½(1/e) times the diameter of the radar antenna, $w_t(z)$ is the diameter of the transmitter beam, and $w_r(z)$ is the diameter of the reflected beam. These latter parameters are determined by substituting ½(1/e) times the diameter of the transmitter and the reflector, respectively. The reflector is assumed to be a circular mirror normal to the direction of propagation of the transmitter beam for these calculations.

## 3. MEASUREMENTS

Measurements of AOA fluctuations over both one-and two-way paths were made at Technovative Applications, Inc. in Brea, CA using an interferometric receiver. The one-way path was 3.5 km in length, and the two-way path was 25 km in length. The experimental configuration with the interferometric radar used for measuring the one-way fluctuations is shown in Figure 1. Figure 2 shows the results of calculating the AOA as a function of range using Equation (1) for five different situations. Figure 3 shows the measured values of AOA. The colors in Figure 3 correspond to those in Figure 2 for a given value of $C_n^2$. The values of $C_n^2$ were determined by measuring the amplitude fluctuations and calculating this parameter using the equation

$$\sigma_\chi^2 = 0.31 C_n^2 k^{7/6} L^{11/6}, \tag{4}$$

where $\sigma_\chi^2$ is log amplitude variance, k is wavenumber, and L is path length. Values of AOA calculated by the geometrical optics method described in [1], denoted by GO, as well as those calculated by a numerical solution to Equation (1), denoted by PO, are shown. In making this latter calculation, the beamwidth given by Equation [2]

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| 1. REPORT DATE<br>**00 DEC 2004** | | 2. REPORT TYPE<br>**N/A** | | | 3. DATES COVERED<br>**-** |
| 4. TITLE AND SUBTITLE<br>**Theory And Measurements Of Angle-Of-Arrival Of Diffraction-Limited Electromagnetic Wave Beams In The Turbulent Atmosphere** | | | | | 5a. CONTRACT NUMBER<br>5b. GRANT NUMBER<br>5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | | | | | 5d. PROJECT NUMBER<br>5e. TASK NUMBER<br>5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>**US Army Space and Missile Defense Command Huntsville, Alabama 35807-3801; Georgia Institute of Technology Atlanta, Georgia 30332** | | | | | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | | | | | 10. SPONSOR/MONITOR'S ACRONYM(S)<br>11. SPONSOR/MONITOR'S REPORT NUMBER(S) |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br>**Approved for public release, distribution unlimited** | | | | | |
| 13. SUPPLEMENTARY NOTES<br>**See also ADM001736, Proceedings for the Army Science Conference (24th) Held on 29 November - 2 December 2005 in Orlando, Florida. , The original document contains color images.** | | | | | |
| 14. ABSTRACT | | | | | |
| 15. SUBJECT TERMS | | | | | |
| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
| a. REPORT<br>**unclassified** | b. ABSTRACT<br>**unclassified** | c. THIS PAGE<br>**unclassified** | **UU** | **2** | |

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was used. The agreement is considered good for this type of experiment.

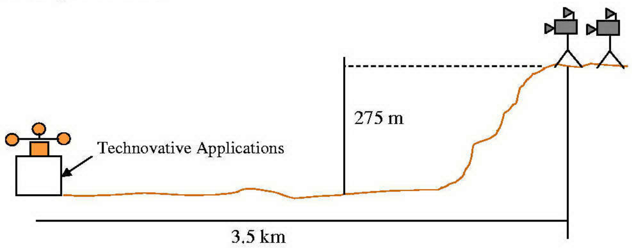


Figure 1. Experimental configuration used for one-way measurements.

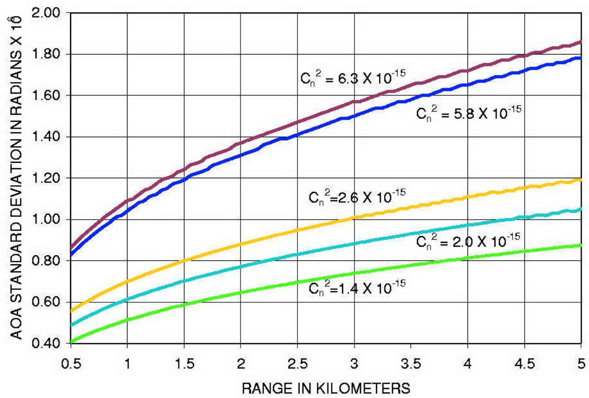


Figure 2. Calculated AOA variations for the one-way path. The colors correspond to the different values of $C_n^2$ as shown in Figure 3.

We have also measured AOA fluctuations over a two-way path length of 25 km. In this case, a passive reflector was used as the target rather than the active repeaters used for the one-way measurements. A scatter plot showing the results of these measurements is shown in Figure 4. The measured standard deviation of these fluctuations is 10 microradians. Calculation of this standard deviation using Equation 3 and a $C_n^2$ value of 3.3 X $10^{-13}$ $m^{-2/3}$ gives a value of 27 microradians. This level of agreement is considered fair.

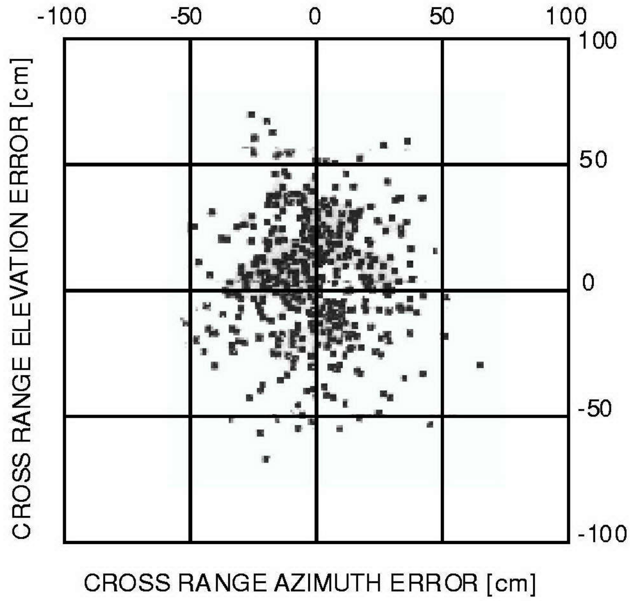


Figure 4. AOA fluctuations over two-way path of 25 km.

## 4. CONCLUSIONS

In this paper, we have described development of a theory of AOA and measurements of this phenomenon. Agreement between theory and measurements is good.

## REFERENCES


[1] J. H. Churnside and R. J. Lataitis, "Angle of Arrival Fluctuations of a Reflected Beam in Atmospheric Turbulence", *J. Opt. Soc. Am.* 4, July 1987, 1264.

[2] R. W. McMillan, et al. "Angle of Arrival of a Radar Beam in Atmospheric Turbulence", Proceedings of IEEE Radar Conference '02, Long Beach, CA, April 2002.

[3] R. W. McMillan and G. L. Freeman, Jr., "Angle of Arrival of a Focused Gaussian Beam in Atmospheric Turbulence", Fourth International Symposium on the Physics and Engineering of Millimeter Waves, Kharkov, Ukraine, June 2001.

[4] A. Yariv, *Quantum Electronics*, 3rd Edition, New York, John Wiley, 1989, Chapter 6.


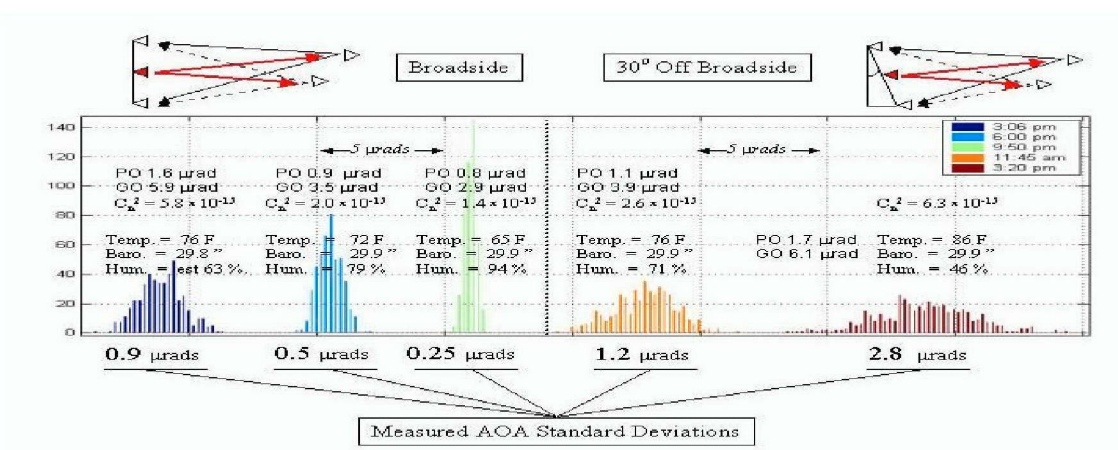


Figure 3. Calculated and measured values of AOA for a 3.5 km one-way path.